

Stars 1

Star parallax See Kirk p.126

In the following $x(\text{unit})$ means the number for x in these units.



$$d(\text{AU}) \cdot \theta(\text{rad}) = 1 \text{ (arc of circle length 1AU)}$$

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m.}$$

$$\text{So } d(\text{AU}) = \frac{1}{\theta(\text{rad})}$$

We now invent a new distance unit: pc.

so that $d(\text{pc}) = \frac{1}{\theta(\text{''})}$ where $1'' = \frac{1}{3600}^\circ$ (arcsec)

We want to know how to get from pc to AU:

$$k d(\text{pc}) = d(\text{AU}).$$

Where

$$\theta(\text{''}) = 3600 \times \frac{180^\circ}{\pi} \theta(\text{rad})$$

$$\text{So: } k = \frac{d(\text{AU})}{d(\text{pc})} = \frac{\theta(\text{''})}{\theta(\text{rad})} = 3600 \times \frac{180}{\pi}$$

$$\text{or } k = 206265$$

$$\text{so: } 1 \text{ pc} = 206265 \text{ AU} = \text{~~4~~ 3.26 ly.}$$

Minimum measurable $\theta = 0.01'' \Rightarrow$

Maximum measurable dist. = 100 pc.

(See eventually Stjerner.pdf, drawings p.2).

Luminosity, brightness, magnitude (See Kirk p. 122, 127)

Luminosity (absolute) [W]: Total power radiated from star.

Brightness (apparent) [W m^{-2}]: Intensity of starlight at Earth.

$$(*)1) \quad b = \frac{L}{4\pi d^2}, \quad d \text{ distance to star}$$

Apparent magnitude: Logarithmic measure of brightness.

Comparing two stars, 1 & 2:

$$(*)2) \quad m_2 - m_1 = -\frac{5}{2} \log\left(\frac{b_2}{b_1}\right)$$

If $m_2 - m_1 = 5$, then $\frac{b_2}{b_1} = 10^{-2}$ (OBS! $m_2 > m_1 \Leftrightarrow b_2 < b_1$)

If $m_2 - m_1 = -1$, then $\frac{b_2}{b_1} = 10^{\frac{2}{5}} = 2.512$.

(The zero-point of the m-scale traditionally was defined, first so $m=6$ correspond to faintest observable star w. naked eye, later so $m=0$ for the star Vega, now more modern definition).

Absolute magnitude M

	Real position	Virtual position
Dist.	d	10pc
Magni-tude	m	M
Brightness	b	b_{10pc}

Definition: The absolute magnitude M for a star is the apparent magnitude it would have if positioned at dist. 10pc.

From (*)1) and (*)2):

$$m - M = -\frac{5}{2} \log\left(\frac{b}{b_{10pc}}\right) \quad \& \quad \frac{b}{b_{10pc}} = \frac{10^2}{d^2}$$

$$\therefore m - M = -\frac{5}{2} \log\left(\frac{10^2}{d^2}\right) = 5 \log\left(\frac{d}{10}\right)$$

$$\text{so } m - M = 5 \log\left(\frac{d}{10}\right) = 5 \log d - 5 \quad (d \text{ in pc})$$

Star 3

Luminosity & absolute magnitude M

Compare an arbitrary star (*) with the Sun (s).

$$(*)2) \quad m_* - m_s = -\frac{5}{2} \log\left(\frac{b_*}{b_s}\right)$$

Now imagine we move the star * and the Sun s to dist. 10pc.

$$\text{then: } (*)3) \quad M_* - M_s = -\frac{5}{2} \log\left(\frac{b_* 10pc}{b_s 10pc}\right)$$

The luminosity $L = 4\pi d^2 b$ (*)1) is of course independent of the position of the star. At dist. 10pc we get:

$$\left\{ \begin{array}{l} L_* = 4\pi (10pc)^2 b_* 10pc \\ L_s = 4\pi (10pc)^2 b_s 10pc \end{array} \right\} \text{ or } \frac{L_*}{L_s} = \frac{b_* 10pc}{b_s 10pc}$$

Inserting this in (*)3) we get:

$$(*)4) \quad \boxed{M_* - M_s = -\frac{5}{2} \log\left(\frac{L_*}{L_s}\right)}$$

Compare with (*)2): apparent magnitude \rightarrow abs. magnitude
brightness \rightarrow luminosity

Luminosity, temperature, wavelength

Star 4

For a star with radius R , temperature T and radiation wavelength λ we have (Kirk p. 74):

Stefan-Boltzmann law: $L = \sigma AT^4 = 4\pi R^2 \sigma T^4$

(σT^4 [Wm^{-2}] is also called the radiation flux at the (F) surface of the star. Compare with brightness b being the radiation flux from the star at the Earth).

$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$$

Wien's displacement law: $\lambda_{\text{max}} \cdot T = 2.90 \times 10^{-3} \text{ m} \cdot \text{K}$

Data compared to Sun The sun is often used as reference for star data

Sun: $L = 3.9 \times 10^{26} \text{ W}$. $T = 5800 \text{ K}$. $\lambda_{\text{max}} = 5.0 \times 10^{-7} \text{ m}$
 $R = 6.96 \times 10^8 \text{ m}$. $M = 4.8$ $F = \sigma T^4 = 6.42 \times 10^7 \text{ Wm}^{-2}$
mass $m = 2.00 \times 10^{30} \text{ kg}$. Spectral class: G2. Lifetime: 10^{10} y

Mass-luminosity relation & lifetime of star.

Main sequence stars (in HR diagram) on the average follow an empirical mass-luminosity relation: $L \propto m^{3.5}$

The amount of fuel $\propto m$.

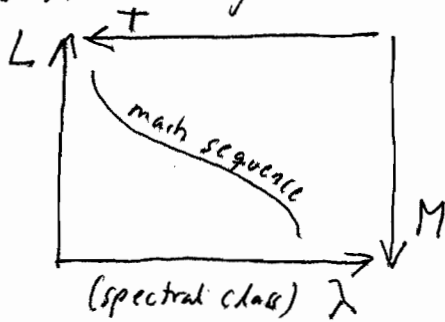
Lifetime $\tau \propto m$ and $\tau \propto \frac{1}{L}$.

Average lifetime: $\tau_* = 10^{10} \text{ y} \frac{M_*}{M_s} \times \frac{L_s}{L_*} = 10^{10} \text{ y} \left(\frac{M_s}{M_*} \right)^{2.5}$

The H-R diagram (Kirk p.125)

As $M \leftrightarrow L$ (p.3 *4) and $T \leftrightarrow \lambda$ (p.4, Wien)

the H-R diagram can be presented in several ways.



Assuming the star to be a main sequence star, H-R-diagram relates L to spectral class ("spectroscopic parallax")

Distance and knowledge of star

See strategy-charts 1 & 2 from "Physics" by Gregg Kerr and Paul Roth, IBID Press.

Method	Range	Remark
Trigonometry	Solar system	
Parallax	< 100 pc	See chart 1
"Spectroscopic" parallax	< 10 Mpc	See chart 2
Cepheid variables	< 60 Mpc (?)	See chart 2 Problem: Two types of Ceph.
Super redgiants, super blue giants, supernovae	< 250 Mpc	
Global clusters, supernovae	< 900 Mpc	
Supernovae	---	
Redshift	"edge" of Universe	Hubble's law (Kirk p.135)

Also lots of other special methods exists!

Hyade method, Tully-Fisher relation, ---

Chart 1 Distance measured by parallax:

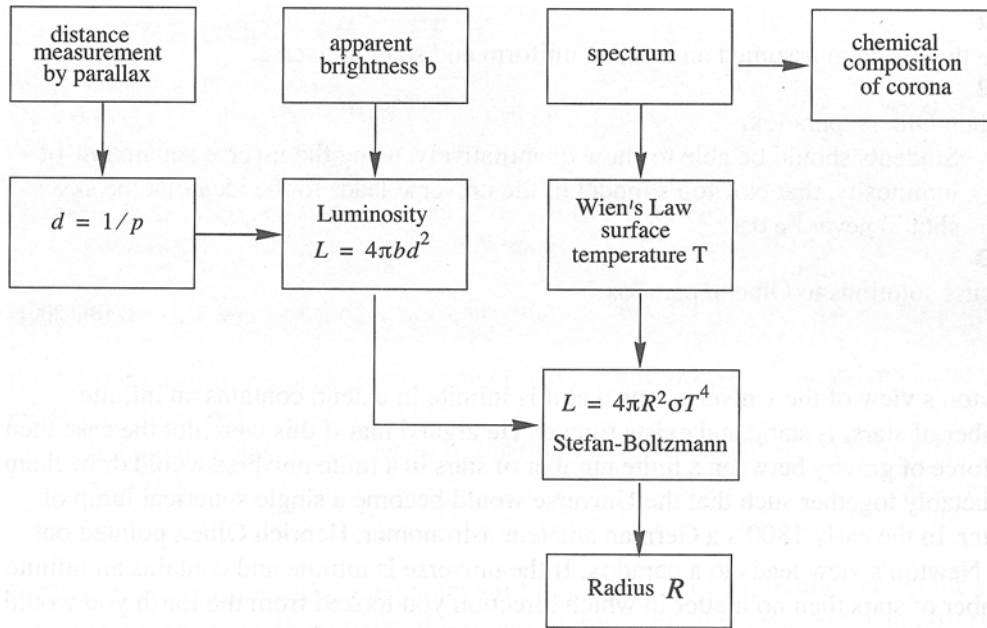


Chart 2 Distance determined by spectroscopic parallax/cepheid variables.

