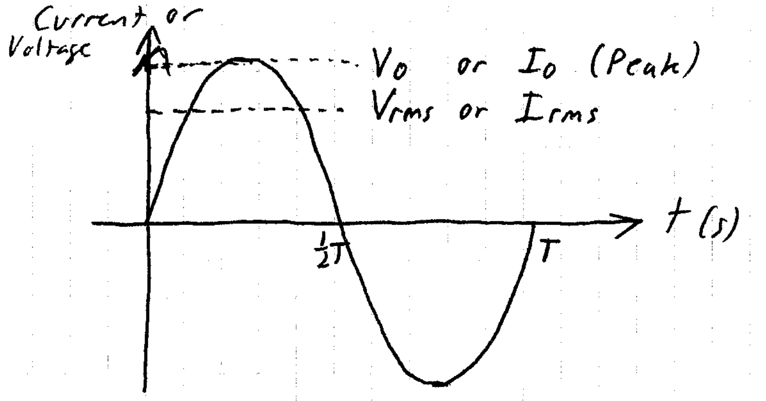
Alternating Current (AC)



$V = V_0 \sin(\omega t)$
$I = I_0 \sin(\omega t)$
$\omega = 2\pi f$, $f = \frac{1}{T}$
V_0: Peak voltage
I_0: Peak current

Time-average is defined as: $\overline{f(t)} = \frac{1}{T}\int_0^T f(t)\,dt.$ (*)

$\overline{V} = 0$ and $\overline{I} = 0$ because $\frac{1}{T}\int_0^T \sin(\omega t)\,dt = 0.$
It makes more sense then to define the <u>rms-value</u> (rms = "root-mean-square") or <u>effective values</u>:

$$\boxed{\begin{aligned} V_{rms} &= \sqrt{\overline{V^2}} = \frac{1}{\sqrt{2}}V_0 \\ \text{and } I_{rms} &= \sqrt{\overline{I^2}} = \frac{1}{\sqrt{2}}I_0 \end{aligned}}$$

~~and~~ because $\frac{1}{T}\int_0^T \sin^2(\omega t)\,dt = \frac{1}{2}.$

<u>Power</u> (for pure resistors +)

$P = RI^2 = RI_0^2\sin^2(\omega t).$ $\quad \overline{P} = \frac{1}{2}RI_0^2 = R \cdot I_{rms}^2$

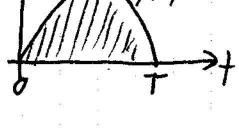
$P = \frac{V^2}{R} = \frac{1}{R}V_0^2\sin^2(\omega t).$ $\quad \overline{P} = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}$

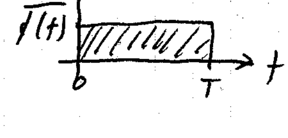
$P = V \cdot I = V_0 I_0 \sin^2(\omega t).$ $\quad \overline{P} = \frac{1}{2}V_0 I_0 = V_{rms} I_{rms}$

– that is, using rms-values (effective values) of V and I the formulas connecting $\overline{P}$, V_{rms} and I_{rms} becomes identical with the DC-formulas.

+ [$\overline{P}$-formulas here <u>only</u> for resistors of the <u>pure</u> type – see Extra p.2]

(*) As $T\,\overline{f(t)} = \int_0^T f(t)\,dt$ the time-average can be illustrated by <u>same areas</u>:





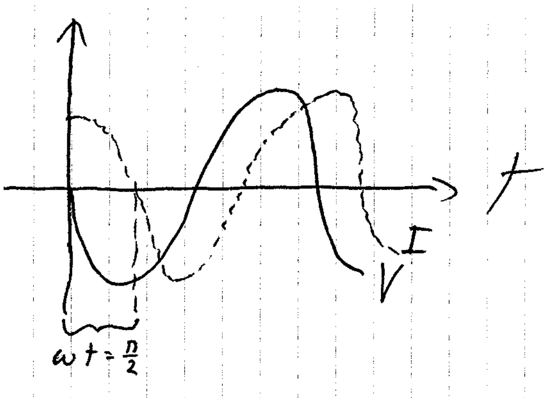
Pure resistance, Ohm's law for AC-circuits

If the voltage and current of the component alternates <u>in phase</u>, that is: $V = V_0 \sin(\omega t)$ and $I = I_0 \sin(\omega t)$ then the component is a pure resistor with resistance R, and $V = RI$ or $V_0 = R I_0$ or $V_{rms} = R I_{rms}$.
The power dissipated becomes: $\overline{P} = V_{rms} I_{rms}$.

Extra: Reactive components

For some components there will be a <u>phase-shift</u> between V and I: $V = V_0 \sin(\omega t)$, $I = I_0 \sin(\omega t - \varphi)$.
Then it can be proved (using $\sin(\omega t - \varphi) = \sin(\omega t)\cos(\varphi) - \cos(\omega t)\sin(\varphi)$) that $\overline{P} = V_{rms} I_{rms} \cos\varphi$.

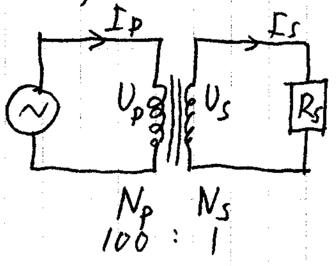
For a coil of wire (an inductor) $\varphi = \frac{\pi}{2}$ so that the current lags behind the voltage in time by a quater of a cycle. The Power becomes: $\overline{P} = 0$, that is, no energy is dissipated in an ideal inductor.



When delivering AC-electricity, the power-plant must adjust its output so that the $V_{rms} I_{rms}$ <u>and</u> the φ match the consumption-line.

Giancoli 3. ed: 18-7, IBID 9.8

Transformation (Giancoli 21-7)



Faraday:

$$\left.\begin{aligned} \frac{V_s}{V_p} &= \frac{N_s}{N_p}. \\ V_p I_p &= V_s I_s \end{aligned}\right\} \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}$$

No power-loss

From the primary side the resistance R_s creates the resistance:

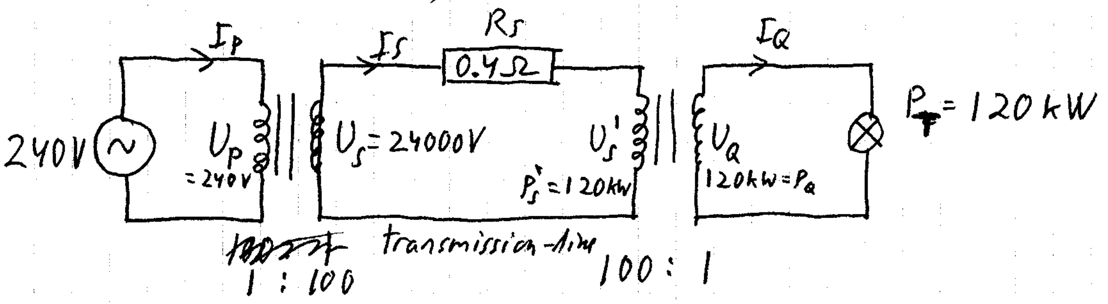
$$R_p = \frac{V_p}{I_p} = V_p \times \frac{1}{I_p} = \frac{N_p}{N_s} V_s \times \frac{N_p}{N_s}\frac{1}{I_s} = \left(\frac{N_p}{N_s}\right)^2 \frac{V_s}{I_s}$$

that is: $R_p = \left(\frac{N_p}{N_s}\right)^2 R_s$.

Example: $\frac{N_p}{N_s} = 100$, $R_s = 0.1\Omega$. Then $R_p = 1000\Omega$

Transmission of power by step-up and step-down Volt.

(Giancoli Ex. 21-10).



Power in transmission-line:

$24000V \times I_s = 0.4\Omega \cdot I_s^2 + 120kW$. Solve for I_s.

Assuming $0.4\Omega \times I_s^2 << 120kW$: $I_s = \frac{120kW}{24000V} = \underline{5A}$

Power-loss in transmission-line: $0.4\Omega \times I_s^2 = \underline{10W}$.

(Usable solution of the quadratic equation: $I_s = 5.00042A$)

Power-loss without step-up, step-down:

$I \approx \frac{120kW}{240V} = 500A$. $\quad 0.4\Omega \times I^2 = \underline{100kW}$