What is this thing called electricity?

1. Electric charge

Names: electric charge, charge.
Symbols of physical entities: Q, q, ΔQ.
SI-unit: Coulomb, C.

Definition: 1 C is the charge of $6{,}24 \cdot 10^{18}$ protons.

Physical constants:
The fundamental charge: $e = 1{,}602 \cdot 10^{-19}$ C.
Charge of proton: e. Charge of electron: –e.

Phenomena: Static electricity.

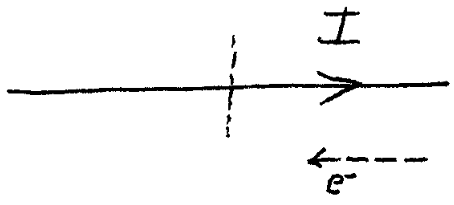
Analogy: A water molecule.

Rules: In an electrical isolated system the total charge is conserved.

2. Electric current

Names: electric current, current.
Symbols of physical entities: I, i.
SI-unit: Amperes, A.

Definition: The current I is charge ΔQ per time Δt passing a cross section of the wire:

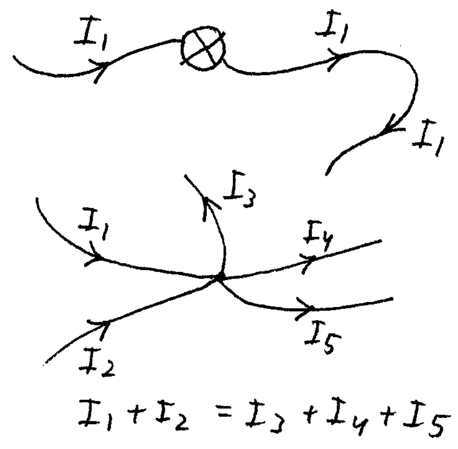


$$I = \frac{\Delta Q}{\Delta t} \quad (1\ \text{A} = 1\ \text{C/s} = 1\ \text{C s}^{-1})$$

The direction of the current is defined as the direction of assumed positive charges and is marked with an arrow. In reality the moving charged particles (charge carriers) that make up the electric current are electrons moving in the opposite direction (as the electrons carries a negative charge).

Phenomena: It is the movement of free electrons in a conductor that make up the electric current. A metal is a conductor. Substances without free electrons are called isolators. Some substances have a special lattice-structure in which the number of free electrons can vary. Such substances are called semi-conductors and they play an important role in electronic devices.

Analogy: Current of water.



Rules: The equation of definition can be changed to:

$$I = \frac{\Delta Q}{\Delta t} \quad \Leftrightarrow \quad \Delta Q = I \cdot dt \quad \Leftrightarrow \quad \Delta t = \frac{\Delta Q}{I}$$

Rules of current in electrical circuits (Kirchoff's laws):

1. The current is the same in any point of an undivided conducting wire (with or without electrical components).

2. The total current arriving at a junction in a circuit must equal the total current leaving the junction.

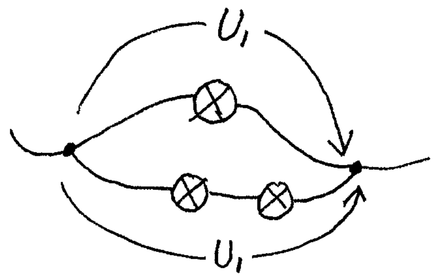
Argumentation: The electric charge is conserved and the current is stationary. No charges are accumulated anywhere. Compare with a stream of water.

3. Potential difference, voltage drop

Names: potential difference, voltage drop, voltage.
Symbols of physical entities: U
SI-unit: volts, V.

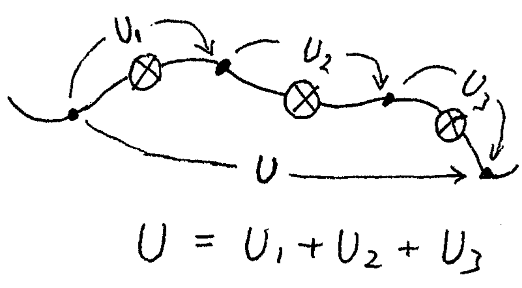
Definition: The potential difference U between two points in a circuit is the energy converted per charge when a charge ΔQ moves between the two points.

$$U = \frac{\Delta E}{\Delta Q} \quad (1\ \mathrm{V} = 1\ \mathrm{J/C})$$



Phenomena: In a thunderstorm the voltage can vary very much with the height. Large potential differences are created between clouds in different levels and between clouds and the surface of Earth.

Analogy: Differences in levels of height of water. Or better: differences in potential energy per kg water.



Rules: The equation of definition can be changed to:

$$U = \frac{\Delta E}{\Delta Q} \quad \Leftrightarrow \quad \Delta E = U \cdot \Delta Q \quad \Leftrightarrow \quad \Delta Q = \frac{\Delta E}{U}$$

Rules of current in electrical circuits (Kirchoff's laws):

1. The voltage drop between two points in a circuit is independent of the route chosen between the two points.

2. The voltage drop along a route between two points is the sum of the voltage drops of the subsections of the route.

4. Electric power

Names: power.
Symbols of physical entities: P
SI-unit: Watt, W.

Definition: Power P is energy ΔE converted per time Δt.

$$P = \frac{\Delta E}{\Delta t} \quad (1\ \mathrm{W} = 1\ \mathrm{J/s})$$

Rules: When a charge ΔQ passes through a voltage drop U in the time Δt the power of the energy conversion becomes:

$$P = \frac{\Delta E}{\Delta t} = \frac{U \cdot \Delta Q}{\Delta t} = U \cdot \frac{\Delta Q}{\Delta t} = U \cdot I$$

Joule's law: When a current I pass through a voltage drop U in the time Δt the energy conversion becomes:

$$\Delta E = U \cdot I \cdot \Delta t$$

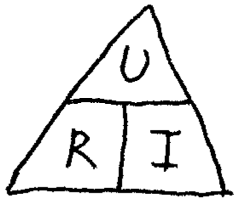
5. Electric resistance

Names: resistance. A resistor is a component with resistance.
Symbols of physical entities: R
SI-unit: Ohm, Ω.

Definition: The resistance of an electric component in a circuit is defined as the voltage drop U between two points just before and just after the component divided by the current I through the component:

$$R = \frac{U}{I} \quad (1\ \Omega\ = 1\ \mathrm{V/A})$$

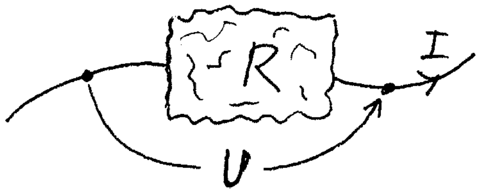
Analogy: Resistance in a water tube. If the water drops from a high to a low level through a tube with resistance, then the stream of water becomes stationary. (Some restrictions using this analogy must be discussed).



Rules: The equation of definition can be changed to:

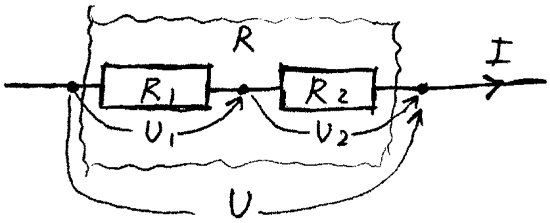
$$R = \frac{U}{I} \quad \Leftrightarrow \quad U = R \cdot I \quad \Leftrightarrow \quad I = \frac{U}{R}$$

This is Ohm's law. The resistance R does not need to be a constant in this relationship. It could for instance depend on I. If the resistance R is a constant then the electric component is called an ohmic conductor. In that case there is proportionality between U and I, and a graph showing I as a function of U (or opposite) will be a straight line through the origin (0,0).

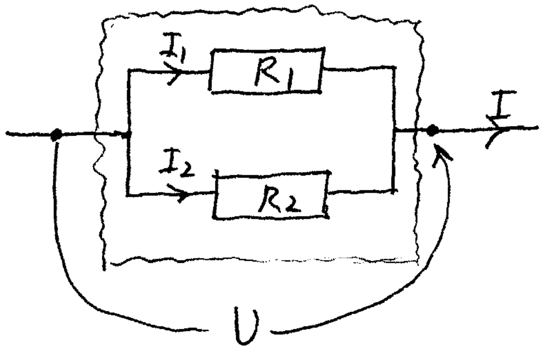


Ohm's law can be applied on one single component or on a section of the circuit consisting of a group of components. In all cases you must observe, that the U in Ohm's law is the voltage drop between two points of the circuit just before and just after the section, and that I is the current through the section. R is the total resistance of the section.

Resistors in series and in parallel. Using Ohm's law and Kirchoff's laws we can set up formulas for the total resistance of a section consisting of two (or more) resistors connected in series and in parallel. The result becomes:



Series: $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$



Parallel: $\frac{1}{\mathrm{R}} = \frac{1}{\mathrm{R}_1} + \frac{1}{\mathrm{R}_2}$

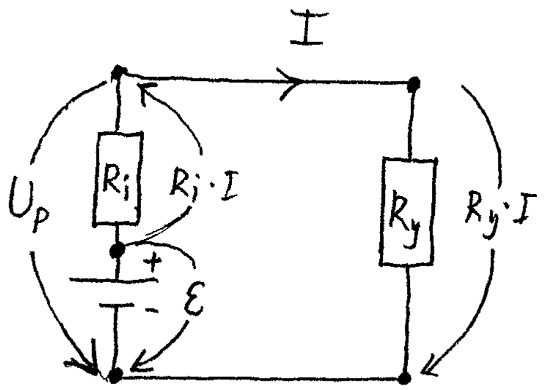
The power of the energy conversion can, using Ohm's law be expressed:

$$\mathrm{P} = \mathrm{U} \cdot \mathrm{I} = \mathrm{R} \cdot \mathrm{I}^2 = \frac{\mathrm{U}^2}{\mathrm{R}}$$

6. Electromotive force

Names: electromotive force, e.m.f.

Symbols of physical entities: $\mathcal{E}$
SI-unit: volts, V.



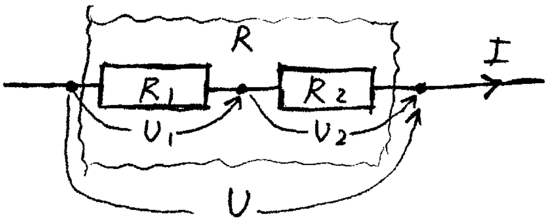
Definition: The current in an electric circuit must be maintained by a source of energy, a power supply, for instance a battery. The e.m.f. $\mathcal{E}$ of the battery is the energy supplied by the battery per passing charge.

Analogy: Water lifted up by an external source of energy, or pressure on a stream of water created by a pump.

Rules: In a circuit consisting of an external resistance R_y and a power supply, for instance a battery with e.m.f. $\mathcal{E}$ and inner resistance R_i, the voltage drop between the poles, the terminal voltage, of the battery U_p becomes:

$$U_p = R_y \cdot I \text{ and } \mathcal{E} = R_y \cdot I + R_i \cdot I \Leftrightarrow U_p = \mathcal{E} - R_i \cdot I$$

7. Resistances in series and parallel



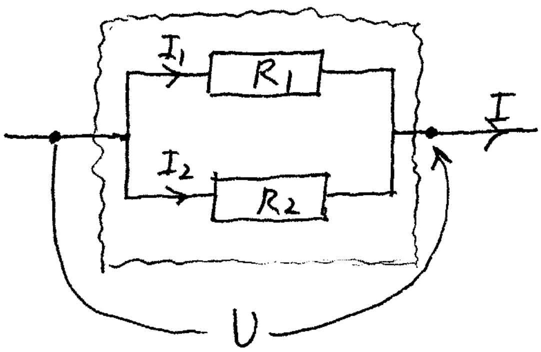
Series:
The section consists of two resistances in series.

Kirchoff's laws:
I is the same through both components R_1 anc R_2.
Sum of voltage drops: $U = U_1 + U_2$.

Ohm's law:
On section: $U = R\,I$
On R1: $U_1 = R_1\,I$
On R2: $U_2 = R_2\,I$

Algebra: $U = U_1 + U_2$
or $R\,I = R_1\,I + R_2\,I$

Conclusion: $R = R_1 + R_2$



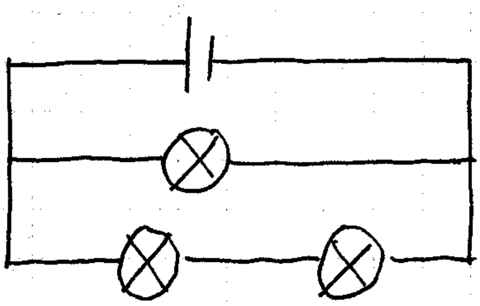
Parallel:
The section consists of two resistances in parallel.

Kirchoff's laws:
U is independent of the route from left to right point.
Sum of currents: $I = I_1 + I_2$.

Ohm's law:
On section: $I = U / R$
On R1: $I_1 = U / R_1$
On R2: $I_2 = U / R_2$

Algebra: $I = I_1 + I_2$
or $U / R = U / R_1 + U / R_2$

Conclusion: $1 / R = 1 / R_1 + 1 / R_2$



Example of application:
Battery with voltage drop between poles: 6 V
Three bulbs, all with resistance 6 Ω.
Total R_{lower} of lower branch: 6 Ω + 6 Ω = 12 Ω
Total R_{total}: $1 / R = 1 / (6\ \Omega) + 1 / (12\ \Omega)$
$= 3 / (12\ \Omega) = 0.25\ \Omega^{-1}$ so $R_{total} = 4\ \Omega$.
Voltage drop of section and both branches: U = 6 V.
Current though section: Itotal = 6 V / 4 Ω = 1.5 A
Current of upper branch: I_{upper} = 6 V / 6 Ω = 1 A
Current of lower branch: I_{lower} = 6 V / 12 Ω = 0.5 A
Check of sum of currents: $I_{total} = I_{upper} + I_{lower}$

8. Examples of circuits

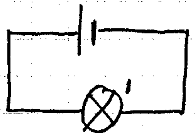
In the 9 circuits below the total voltage drop between the poles of the battery is 6 V and all bulbs have a resistance of 6 Ω.

1. For each circuit:
    a. Find the total resistance
    b. Find the total current
    c. Find the current through and the voltage drop over each bulb

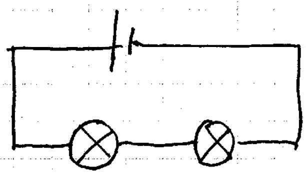
2. Create a circuit containing 5 bulbs (same data as before).
    a. Solve this circuit as in 1.

b. Hand this circuit over to another group and ask them to solve it as in 1.

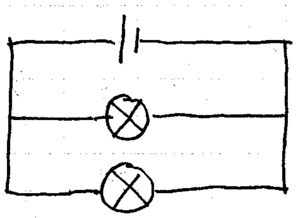
SK1



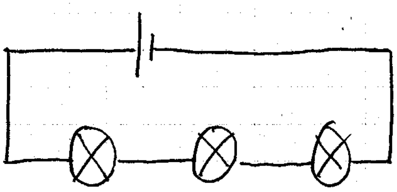
SK2



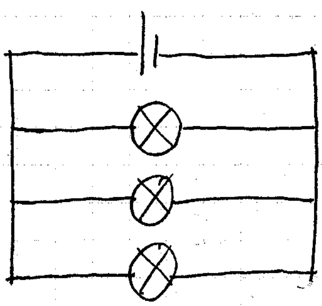
SK3



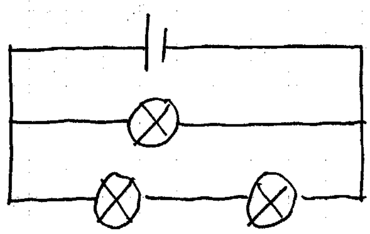
SK4



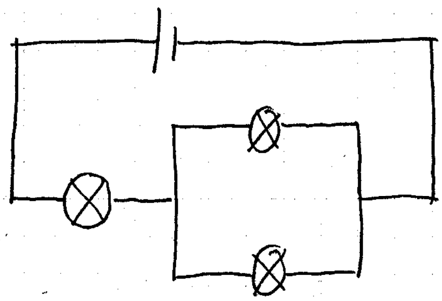
SK5



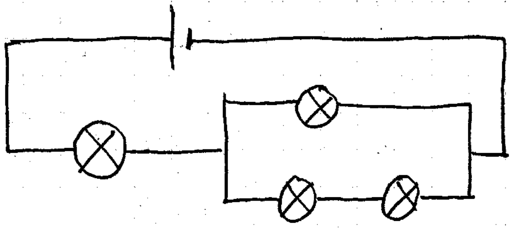
SK6



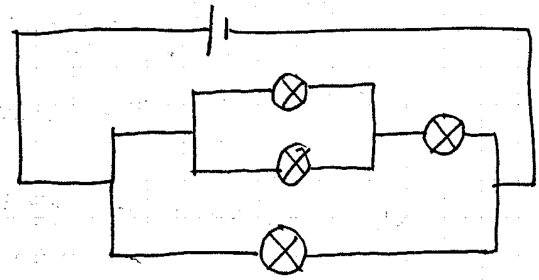
SK7



SK8



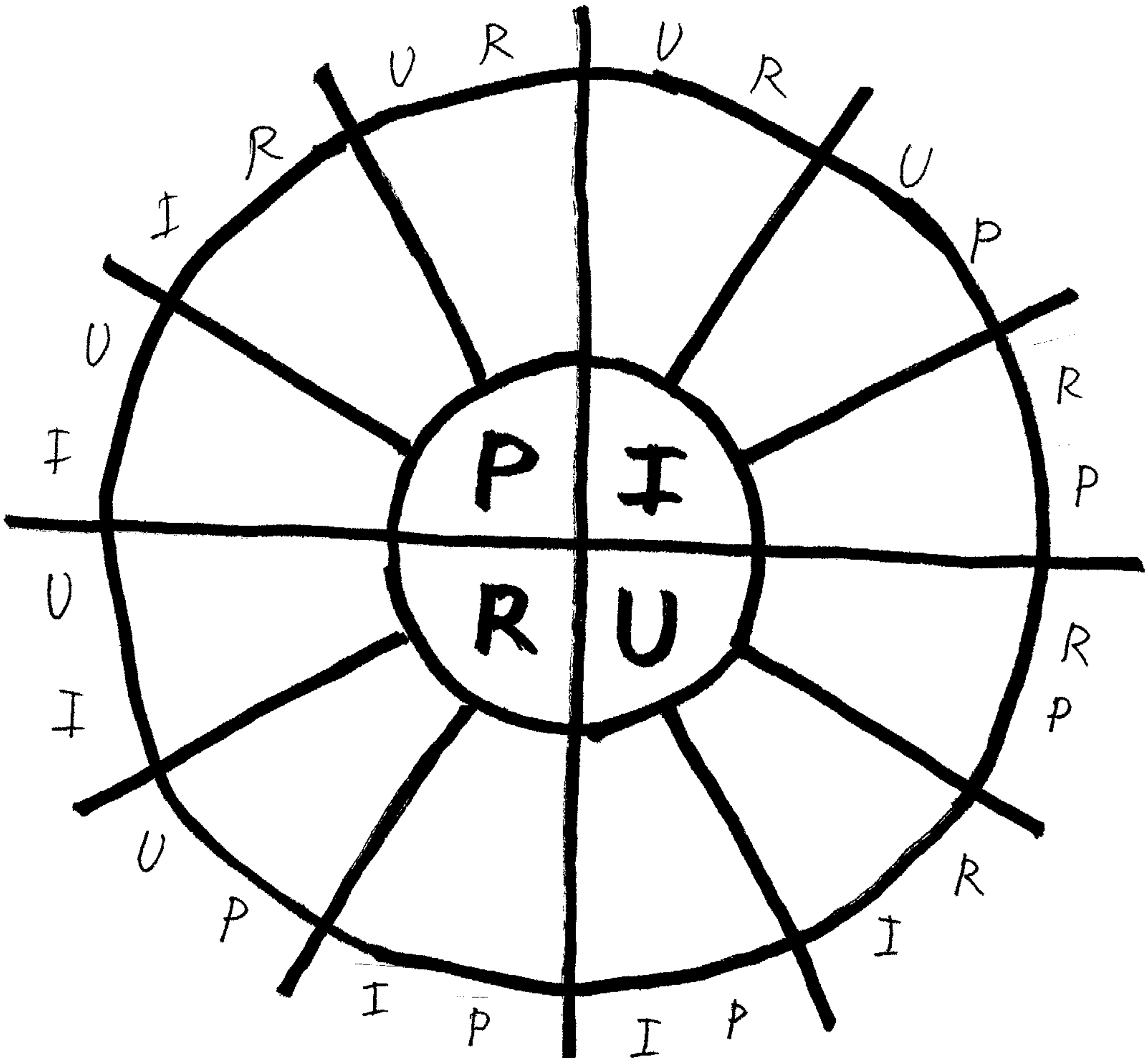
SK9



9. Formulas of electrity

1. Fill out the empty cells with relevant formulas, such that the formula in the empty cell expresses the entity in the corresponding cell of the inner circle (common to three empty cells) using the entities of the corresponding cell of the outer circle.
   For instance: In the cell between "4 o'clock" and "5 o'clock" you should create a formula expressing U using R and I, which of course is: U = R I.

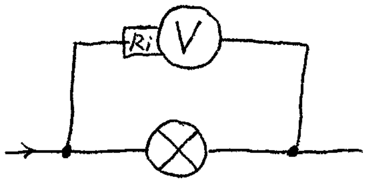
2. Choose a cell and formulate an exercise for which the solution requires direct use of the formula in that cell. Hand this exercise over to another group (without telling which cell to be used).



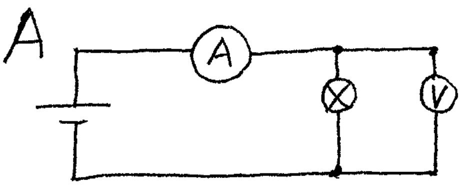
Measurements – el.-components and circuits

1. Measurements in general

The current through an electric component (or a chosen section of a circuit) is measured by use of an ammeter in series with the component. The inner resistance of the ammeter should be low such that the influence on original the circuit is negligible.

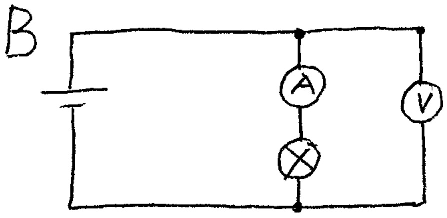


The potential difference – or voltage drop – over a component (or section) is measured by use of a voltmeter in parallel with the component. The inner resistance of the voltmeter should be high such that the influence on the original circuit is negligible.



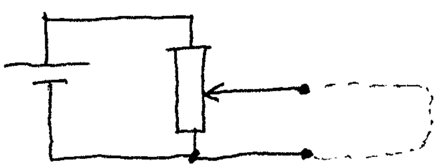
To make a simultaneous measurement of the current through and the voltage drop over a component, one of the following two setups, A or B, can be applied.

The setup A leads to a systematic error, because the ammeter shows the total current through both the component and the voltmeter. This error will be negligible if the inner resistance of the voltmeter is high in relation to the resistance of the component.



The setup B leads to a systematic error, because the voltmeter shows the total voltage drop over both the component and the ammeter. This error will be negligible if the inner resistance of the ammeter is low in relation to the resistance of the component.

In some measurements it is practicable to be able to vary the terminal voltage. This can be done by use of a variable power supply.

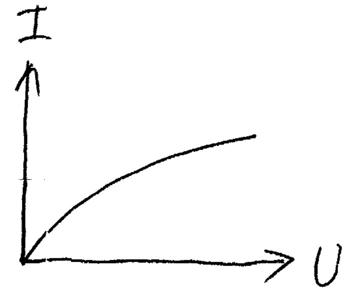


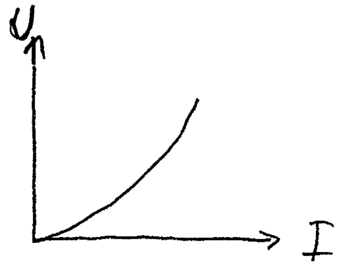
As an alternative the voltage can be varied by use of a potential divider, which is a special circuit setup using a variable resistor.

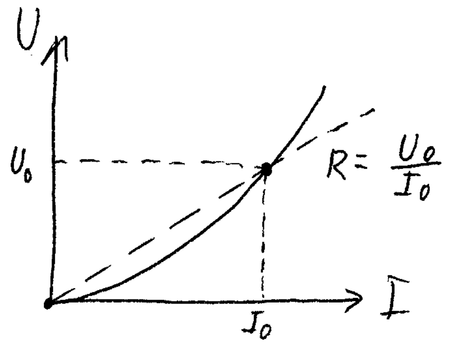
2. Measurements on circuits

We use a power supply and small light bulbs.
Setup the circuits SK1 – SK9 of p. 6. Find currents in the different branches and the voltage drops over the bulbs. Compare with Kirchoff's and Ohm's rules.

3. Component characteristics







The characteristic of an electrical component is a graph showing how I depends on U – a (x,y) = (U,I)-graph with U on the x-axis and I on the y-axis, or how U depends on I – a (I,U)-graph with I on the x-axis and U on the y-axis.

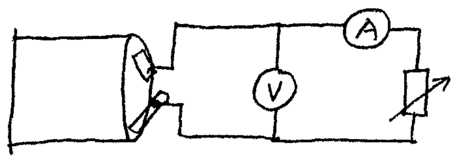
Normally in experimental setups you can vary the terminal voltage, not the current. Then U becomes the independent variable, and then the (U,I)-graph is the mathematically most correct way of showing the dependency. Anyway it is common even in this case to present the characteristic as a (I,U)-graph.

On a (I,U)-graph the resistance R of the component for specific values corresponding to the point (I_o,U_o) is the gradient of the line connecting the origin (0,0) with (I_o,U_o).
[On a (U,I)-graph the gradient of the line from (0,0) to (U_o,I_o) will correspond to 1/R.]

In the experiment you should get characteristics of some electrical components, for instance small light bulbs (max 6V), "normal resistors" (max 2A), metal wires and diodes (max 6V).

Consider which of the setups A or B should be applied in the different cases.

The teacher will maybe show you the characteristic of high voltage light bulbs and carbon filament bulbs.



Also find the characteristic of a battery, that the dependency between the terminal voltage and the current. The result of this can be compared with the theory of part 6., The electromotive force, p.4.

The results should be presented with tables and graphs. The graphs must be carefully described and commented. What does the different graphs tell about the resistance? Judge if the component is ohmic (see part 5, Electric resistance, p.3).

Random and systematic errors must be discussed.

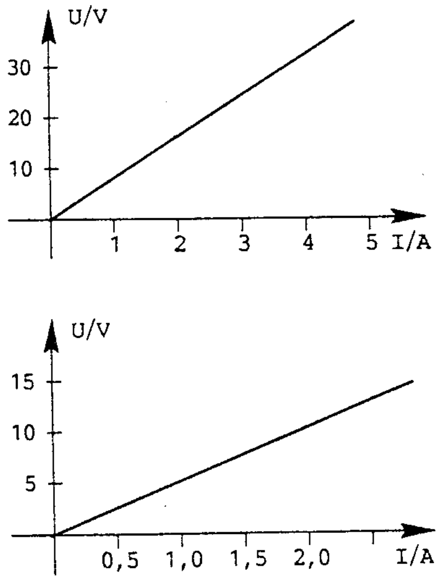
Problems in electrical circuits

1. The current through an electric lamp is 0.27 A and the voltage drop over the lamp is 220 V.
* Find the resistance of the lamp and
* the power transferred.

2. In 2 seconds the current through a lamp is 2 C and the energy transmitted is 440 J.
* Find the voltage drop over the lamp and
* the current through it.

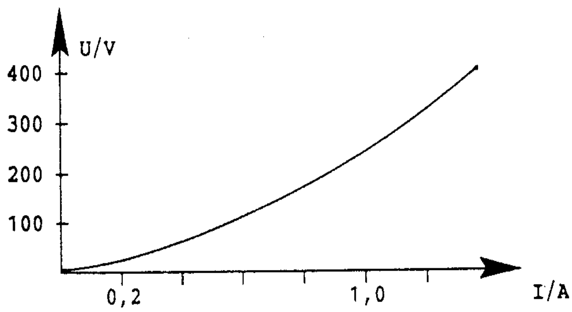
3. A toaster works on a 220 V supply. The current through it is 2.5 A.
Find the power transmitted by the toaster.

4. The starter motor in a car is driven by a 12 V battery. When starting the car 900 W power is transmitted.
* Find the current through the starter and
* its resistance.



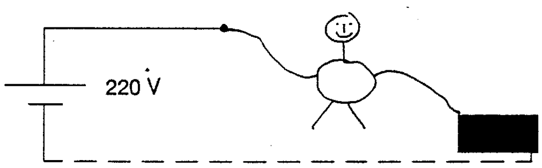
▲ Two graphs for problem 5

5. The graphs show (I,U)-characteristics of two metal wires.
* Are these wires ohmic?
* Find the resistance of each wire.



▲ Graph for problem 6

6. The graph shows (I,U)-characteristic of the filament in lamp.
* Is the filament ohmic?
* Find the resistance of the filament when the current is 0.4 A, 0.8 A and 1.2 A.
* Describe how the resistance is changing when the current increases.



▲ Graph for problem 7

7. The human body has a certain electrical resistance. If you get in contact with electrical equipment or wires such that you become a part of the circuit, a current will pass through parts of your body. If the current passes the heart, a current of 40 mA is enough to kill you. The resistance in your body depends mainly on the humidity of your skin at the points where you get in touch with the el-components. It normally varies between 1 kΩ and 10 kΩ.
* How large a current do you risk to get through you if you touch conducting points with a voltage drop of 220 V?
* Can this current kill you?
* Same questions for a 24 V power supply.

More problems in electrical circuits

1. 10^{19} electrons pass through a wire from A to B in 2 s.
* Which direction and what size does the current I have?

2. In an el-wire for a toaster the current is 2 A.
* How much electric charge passes a cross section of the wire during 5 minutes?
* How many electrons does this correspond to?

3. The current in a wire is 0.02 A.
How much time is required for 1 C to pass a cross section of the wire?

4. A power supply can deliver a current of max. 5 A.
The voltage drop between the poles can vary between 0 and 24 V.
* What is the max. power?

5. A bulb is marked (6V, 30W).
* What does this mean?
* How much current can the bulb take?

6. The cabin-outlet of a car has a max. power of 50 W.
* How much max. energy is spend (in kWh and in J) if it is on for 20 s?
The power supply for the outlet is the car-battery of 12 V.
* How much max. current is delivered?
* What electric charge does this correspond to?

7. A bulb is marked 220 V, 60 W.
* Find the resistance.
* How much power is transferred in the bulb if it is attached to 110 V, assuming the resistance to be unaltered?

8. A fuse is marked 220 V, 10 A.
* How many 60 W bulbs can be connected with the outlet of this fuse?

9. An iron contains a heating wire with a resistance of 60 Ω and is attached to 220 V.

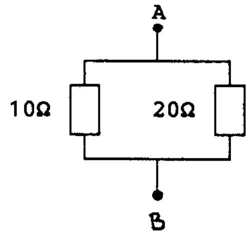
* Find the power of the iron.

The wire is now substituted by a wire with resistance 30 Ω.

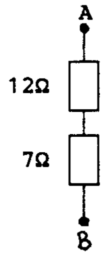
* With what factor is the power changed?

10. Find the total resistance in the circuits shown:

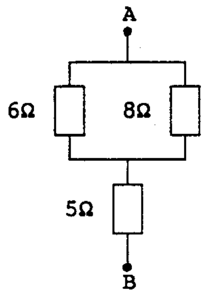
a)



b)



c)



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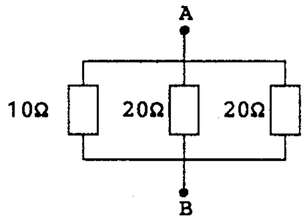

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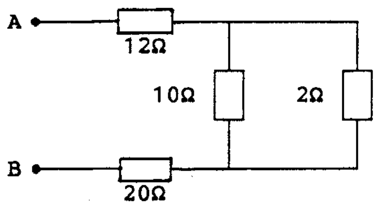

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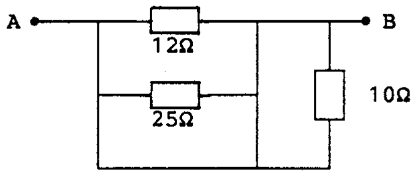
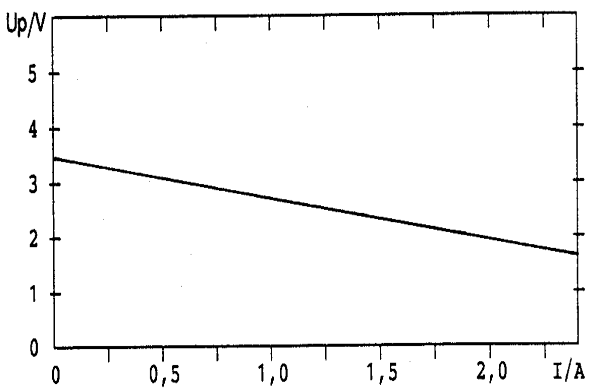




▲ Graph for problem 11

11. The graph shows the characteristic of a power supply (e.g. a battery).

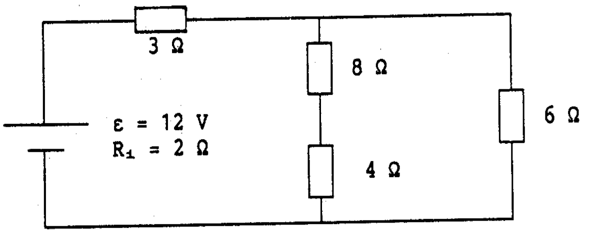
* Find the e.m.f. (ε), the internal resistance (R_i).
* Write an equation of the characteristic.

The power supply delivers power to an electromotor with a current of 1.5 A.

* Find the voltage drop between the poles (U_p).
* Find the voltage drop over the inner resistance.
* Find the total power delivered by the power supply (ε I)
* Find the power delivered in the external resistance.

The efficiency η of the power supply is defined as the power delivered to the external resistance over the total power, in percentage.

* Find the efficiency of the power supply when delivering 1.5 A.



▲ Graph for problem 12

12. In the circuit shown:
* Find the total current and the currents and voltage drops over all components.

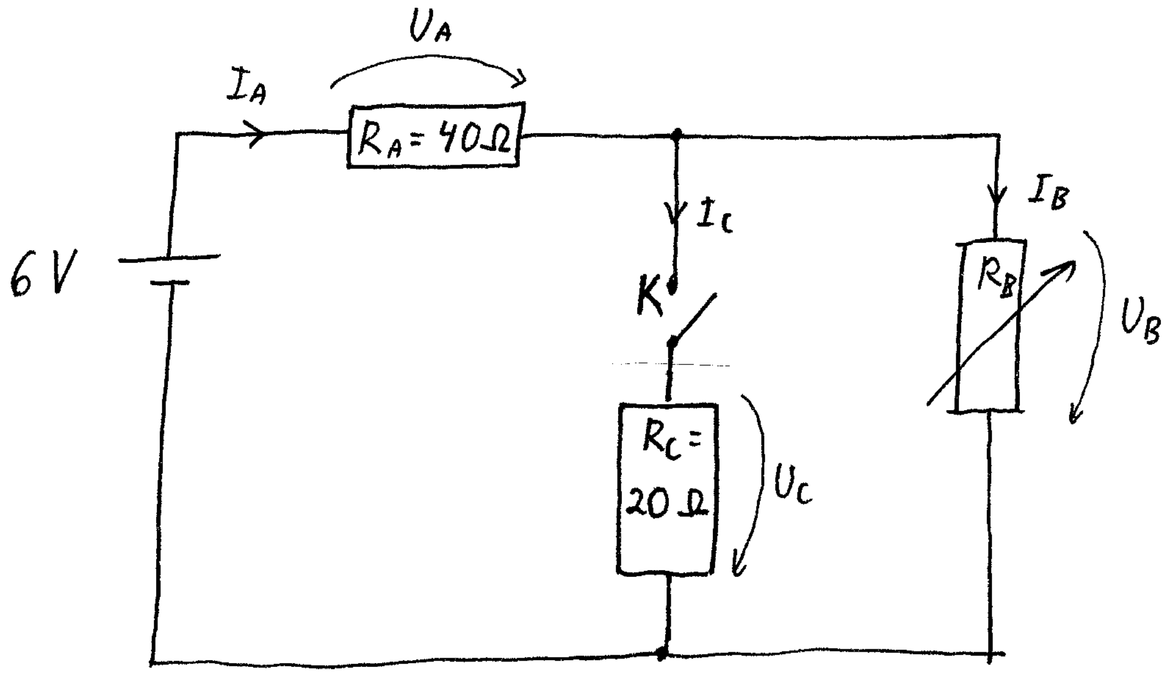
13. A battery with e.m.f. 4.5 V and internal resistance 2 Ω is connected to a bulb which filament has a resistance of 5 Ω.
* Find the total current.
* Find the power supplied to the bulb.
* Find the efficiency η.

14. The voltage drop between the poles of an outlet is measured to be 226 V when not loaded. Then a 2000 W heater is connected, and now the voltage drop between the poles of the outlet is measured to be 222 V.
* Find the total current.
* Find the inner resistance of the outlet.

15. A bulb is marked 4.2 V and 0.2 A. The bulb is connected to a 4.5 V battery with internal resistance 3 Ω.
* Find the resistance of the bulb.
* Find the current though the bulb.
* Find the voltage drop over the bulb.
* Find the power transformed in the bulb.
* Find the power transformed in the inner resistance of the battery.
* Find the efficiency η.

16. A power supply has e.m.f. of 10 V and inner resistance of 4 Ω. In an experiment the supply is loaded with a variable resistor giving an external resistance R_y from 2 Ω to 7 Ω.
* Create a table with columns for external resistance R_y, polar voltage drop U_p, current I, power transformed in the external resistance, P_y and efficiency η.
* Fill out the table with R_y, in steps of 0.5 Ω from 2 Ω to 7 Ω.
* Create a graph showing P_y as a function of R_y.
* Which value of R_y results in maximum P_y?
* Extra: Create a formula of P_y as a function of R_y (containing some constants) and make a plot with your graphic display calculator – then finding the maximum.
* Extra: You could also differentiate the function, then showing the general result: What value should R_y have to give maximum power?

More problems in electrical circuits (2)



The shown circuit contains a power supply (assumed inner resistance 0 Ω) and 3 resistors. The resistor B is variable. The resistor C is in serial with a switch K.

1. The switch K is open. Resistor B is set on 60 Ω.
* Find the total resistance of the circuit.
* Find the current through and the voltage drop over A and B.
* Find the power transmitted in A.

2. The switch K is closed. Resistor B is set on 60 Ω.
* Find the total resistance of the circuit.
* Find the current through and the voltage drop over A, B and C.
* Find the power transmitted in A.

3. The switch K is closed. Resistor B is set on 30 Ω.
* Find the total resistance of the circuit.
* Find the current through and the voltage drop over A, B and C.
* Find the power transmitted in A.

4. The switch K is closed. A marking on resistor A shows that this component at most can take a maximum power of 0.5 W.
* Find the current through and the voltage drop over A with this max. power.
* Find the minimum resistance you can set B to without spoiling component A.