Fields/Gravity

Chan Kee-Lin Steven, pundit@jcphysics.com

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1 Law of universal gravitation

Tool: Pressure in a static fluid
Category: Fields/Gravity
Type: Theorems and Laws

Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distances apart.

If the Law is expressed mathematically,

$$\begin{aligned} F &\propto \frac{m_1 m_2}{r^2} \\ F &= -\frac{G m_1 m_2}{r^2} \qquad (1) \end{aligned}$$

F is the force of attraction.
m_i is the mass of particle i.
r is the distance between particles.
G is the *universal gravitational constant.*

To signify that gravity is always 'attractive', F is always negative.

2 Gravitational Field

Tool: Gravitational Field
Category: Fields/Gravity
Type: Definition

A gravitational field is a region of space where a mass, m, will experience a force. The field is 'caused' by the presence of another mass, M, and is measured in force per unit mass (see figure 1).

$$\text{Field Strength} = -\frac{GM}{r^2} \qquad (2)$$

M is the mass of the body 'causing' the gravitational field.
r is the distance away from mass, M, where the field strength is being measured/calculated.
G is the *universal gravitational constant.*

To find the force exerted by the field on a mass, m, just multiply m by the field strength at that point.

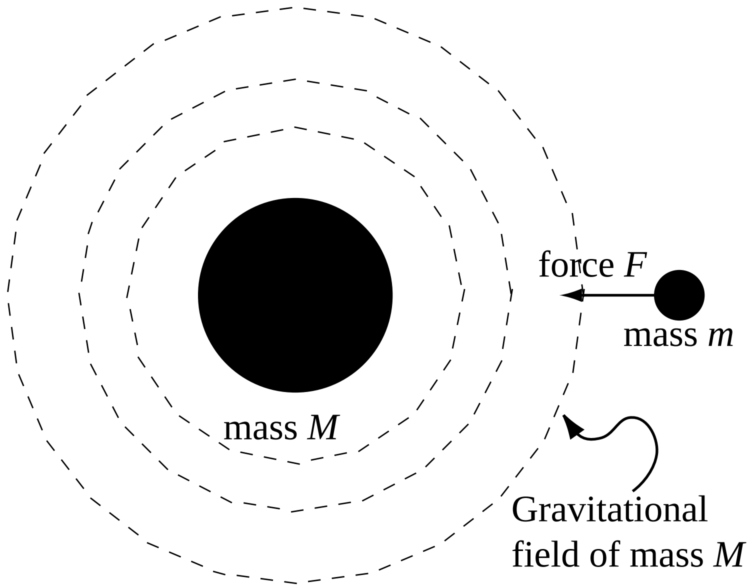

Figure 1: A gravitational field.

3 Variation of gravitational acceleration for the Earth

Tool:	Variation of gravitational acceleration for the Earth
Category:	Fields/Gravity
Type:	Property

On the surface of the earth and beyond, the gravitational acceleration due to the earth varies according to eqn. (2). What about within the earth?

When thinking about the gravitational acceleration inside the earth, it is convenient to divide the earth into 2 pieces. One piece is just a smaller version of the earth (part A) and the other a spherical shell (part B). See figure 2 for details.

Any mass a distance r from the earth's center will feel a attraction from part A and B. However, it is a little known fact that the gravitational force inside a spherical shell (part B) is *zero*. This drastically simplifies the analysis because part B can be ignored. Forget about it.

The mass of part A varies with r and is

$$M = \text{density} \times \text{volume}$$

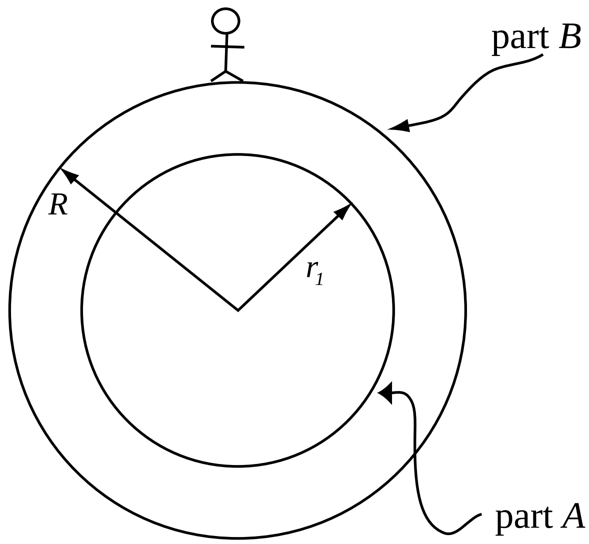

Figure 2: Earth divided into a smaller sphere an larger spherical shell.

$$= \rho_{earth} \times \frac{4}{3}\pi r^3 \tag{3}$$

M is the mass of part A.
ρ_{earth} is the density of the earth. Assumed constant.
r is the distance from the center of the earth.

Eliminating M from eqn. (2) by substituting in eqn. (3) results in

$$\begin{aligned} \text{Field Strength} &= -\frac{GM}{r^2} \\ &= -\frac{G\rho_{earth} \times \frac{4}{3}\pi r^3}{r^2} \\ &\propto r \end{aligned} \tag{4}$$

G is the *universal gravitational constant.*

Therefore, gravity varies linearly with the distance, r, from the center of the earth. Please note: the minus sign in front of eqn. (4) is being ignored in this case. Of course, at the surface of the earth, gravity will resumes the inverse square law relation.

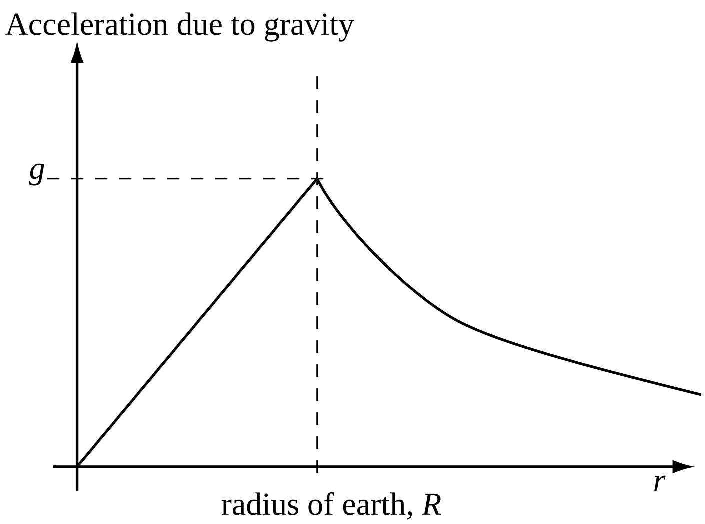

Figure 3: Variation of gravity with distance from the center of the earth.

4 Gravitational Potential

Tool: Gravitational Potential
Category: Fields/Gravity

The gravitational potential at a distance r from a body is the work done by an *external agent* in bringing a unit mass from an infinite distance away to that distance r. For practical purposes, an infinite distance away doesn't really have to be all that far.

The gravitational field strength of the earth diminishes to one-hundredth of its surface value at about 60,000 km. That may seem like a lot until one realizes that 60,000 km is only about one-sixth the distance from the earth to the moon. In astronomical terms, that's miniscule.

Taking the external agent to be the system, then work done on the surroundings (unit mass) by the system is negative. Therefore the gravitational potential is

$$\begin{aligned}\phi &= -\int_{\infty}^{r} F dr' \\ &= -\int_{\infty}^{r} -\frac{GM(1)}{r'^2} dr' \\ &= \left[-\frac{GM}{r'}\right]_{\infty}^{r}\end{aligned}$$

$$= -\frac{GM}{r} \tag{5}$$

ϕ is the gravitational potential.
r is the distance from the center of the body that is creating this gravity field.
G is the universal gravitational constant.
M is the mass of the body.