Group 4 Proj. "Patricia-group"-and others? 6-5-2008

Climatenotes01.pdf

Ref.: Kirk 2007 p.75.
Be aware of the concept "Surface heat capacity", C_s.

Ref.: Giancoli 2005 p. 400-401

Ref.: Notes on cooling by radiation.

Ref: Web-pages on cooling by radiation.
http://hyperphysics ---

Ref. Tim Kirk & Neil Hodgson, 2007, chap. 19,
which includes some hints on modelling, p. 237 ff.
(from http://climateprediction.net).

Cooling by radiation

- Teapot-example, Giancoli 2005 p. 401

$$\frac{\Delta T}{\Delta t} \rightarrow \frac{dT}{dt} = \frac{1}{mc}\frac{dQ}{dt} = -\frac{1}{mc}\varepsilon\sigma A\left(T^4 - T_a^4\right)$$

Here T is temp. of teapot, T_a temp. of surroundings (ambient temp.)

This is a differential equation – to be solved!

To simplify we assume $T^4 >> T_a^4$ and put $T_a = 0$.

(Ref.: Web-page "Radiation Cooling Time").

So: $\frac{dT}{dt} = -\frac{\varepsilon\sigma A}{mc}T^4$ or $\frac{dT}{T} = -\frac{\varepsilon\sigma A}{mc}dt$

(You are allowed to manipulate with dt and dT like this!)

Integrating on both sides:

$$\int \frac{dT}{T^4} = \int -\frac{\varepsilon\sigma A}{mc}dt \quad \text{or} \quad \int \frac{1}{T^4}dT = -\frac{\varepsilon\sigma A}{mc}\int dt$$

or: $-\frac{1}{3}T^{-3} = -\frac{\varepsilon\sigma A}{mc}t + \text{const.}$

If starting temp. is $T = T_h$ (hot) for $t = 0$

we get that $-\frac{1}{3}T_h^{-3} = \text{const.}$

or: $-\frac{1}{3}T^{-3} = -\frac{\varepsilon\sigma A}{mc}t - \frac{1}{3}T_h^{-3}$

or: $t = \frac{mc}{3\varepsilon\sigma A}\left(T^{-3} - T_h^{-3}\right)$

so cooling time to get temp. down to $T = T_f$ (final):

$$t_{cool} = \frac{mc}{3\varepsilon\sigma A}\left(T_f^{-3} - T_h^{-3}\right).$$

Cooling by radiation.

c is the specific heat capacity: $Q = mc\Delta T$, $c: [J\,kg^{-1}K^{-1}]$.

Assuming that the thermal internal energy can be expressed as: $U = Q = \frac{3}{2}NkT$, N number of molecules (not always so! - see notes "Second law of thermodyn." p. 59) then the specific heat capacity can be expressed as

$c = \frac{3}{2}k$ $[J\,\cancel{K}^{-1}\,\text{numb. of molecules}^{-1}]$ (or as $c = \frac{3}{2}R$ $[J\,\cancel{K}^{-1}\,mol^{-1}]$)

In the eq. of cooling time, you can then substitute mc with $\frac{3}{2}Nk$, getting:

$$t_{cool} = \frac{\cancel{3}Nk}{\cancel{3}\cdot 2\,\varepsilon\sigma A}\left(T_c^{-3} - T_h^{-3}\right)$$

The hot sphere on http://hyperphysics... is massive, containing N molecules.

See also same web-site on Kelvin Cooling Time for the Earth.

If only considering the surface of the planet, you could probably use the concept <u>surface heat capacity</u> mentioned in Kirk 2007 p. 75, and also in Kirk & Hodgson, 2007.